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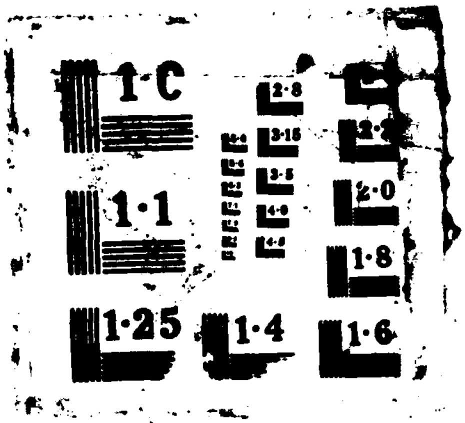

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Transfer matrices and joint coupling matrices are used to compute natural frequencies of vibration of a damaged five-bay planar lattice structure. Seven different states of damage are considered. Each damage state corresponds to a disconnected or partially disconnected joint in the lattice. It is shown that the natural frequencies associated with each damage state are significantly different from the natural frequencies of the undamaged lattice, thus demonstrating that measurement of natural frequencies is a potentially useful method of detecting lattice damage. However, measurement of natural frequencies alone is not sufficient, in general, to determine the location of damage within the lattice. Some suggestions for more complete and quantitative methods of assessing structural integrity of lattice structures are given.

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INTRODUCTION

Successful operation of orbiting space systems requires structures which are effective and reliable in remote and inaccessible environments. Thus, the design of quantitative nondestructive evaluation (NDE) methods is an important part of the reliable implementation of space structures. Ideally, NDE methods for space structures would operate automatically in orbit, would detect any damage or abnormality in the structure, and would provide a quantitative assessment of structural integrity.

In a previous report [1], transfer matrices and joint coupling matrices are used to compute natural frequencies of vibration of a five-bay planar lattice structure. In this report, the problem of detecting damage in the five-bay planar lattice structure is considered. Seven different states of damage are assumed. Each damage state corresponds to a disconnected or partially disconnected joint in the lattice. Transfer matrices and joint coupling matrices are used to compute natural frequencies associated with each damage state. The natural frequencies computed for each damage state are significantly different from the natural frequencies of the undamaged lattice; for example, the frequencies of the first flexible mode of the damaged lattice are 26% to 83% lower than the frequency of the first flexible mode of the undamaged lattice.

The results presented here demonstrate that measurement of natural frequencies is a potentially useful method for detecting damage in lattice structures, at least, for the types of damage considered here. However, it is also shown here that measurement of natural frequencies alone is not sufficient, in general, to determine the location of damage within the lattice structure. Thus, measurement of natural frequencies should be regarded as only a part of a complete NDE method. After the results obtained here are presented, some suggestions for NDE methods which may be capable of providing more quantitative measures of structural integrity are given.

LATTICE MODEL AND DEFINITION OF DAMAGE STATES

The lattice model considered here is shown in Fig. 1. The physical lattice structure which the lattice model of Fig. 1 represents in described in [1]. The members of the lattice of Fig. 1 are assumed to be one-dimensional continua which can extend (and contract) axially and bend flexurally. The members are modelled as classical longitudinal rods for axial motion and as Bernoulli-Euler beams for flexural motion. It is assumed that all motion remains in the plane of the lattice, and that all motion is small. The joints in the lattice model are assumed to be rigid and massless, and to have no spatial extent. The joints are labeled 1 through 12 as shown in Fig. 1.

The damage states considered here are defined as follows. In damage state 1, joint 1 of the lattice is disconnected, as shown in Fig. 2. In damage state 2, joint 3 is completely disconnected, as shown in Fig. 3. In damage state 3, joint 3 is partially disconnected, as shown in Fig. 4. In damage state 4, joint 3 is partially disconnected, as shown in Fig. 5. In damage state 5, joint 5 is completely disconnected, as shown in Fig. 3. In damage state 6, joint 5 is partially disconnected, as shown in Fig. 4. In damage state 7, joint 5 is partially disconnected, as shown in Fig. 5. In each damage state, it is assumed that the lattice structure remains unaltered except for the particular disconnected joint listed above.

NATURAL FREQUENCIES OF DAMAGED LATTICE

The joint coupling matrices for the disconnected and partially disconnected joints in Figs. 2 through 5 are derived in Appendix A. By using these joint coupling matrices in the computer program given in [1], natural frequencies of vibration of the damaged lattice structure may be obtained. The material and geometric constants used in the computation of the natural frequencies are given in [1].

The first twenty-five nonzero natural frequencies of the undamaged lattice and the first twenty-five nonzero natural frequencies associated with each of the seven damage states are given in Tables 1 through 8. The natural frequencies of the undamaged lattice and the natural frequencies associated with damage states 1 and 2 are plotted in Fig. 6. The natural frequencies of the undamaged lattice and the natural frequencies associated with damage states 3, 4 and 5 are plotted in Fig. 7. The natural frequencies of the undamaged lattice and the natural frequencies associated with damage states 6 and 7 are plotted in Fig. 8.

DISCUSSION AND CONCLUDING COMMENTS

The results presented in Tables 1 through 8 show that the natural frequencies associated with each damage state are significantly different from the natural frequencies of the undamaged lattice. For example, the frequencies of the first flexible mode of the damaged lattice are 26% to 83% lower than the frequency of the first flexible mode of the undamaged lattice. Thus, measurement of natural frequencies is a potentially useful method of detecting damage, at least, for the types of damage considered here. Measurement of natural frequencies may be accomplished automatically by detecting peaks in a transfer function, as is commonly done in experimental modal analysis [2].

The large change in natural frequencies due to the disconnected joints considered here reflects the fact that damage or failure of a single joint in a lattice can cause a large change in the overall stiffness of the structure. This sensitivity of lattice structures to failure of a single joint underscores the need for effective NDE methods.

Many damage states are possible, including combinations of the damage states considered here. Experience with a particular structure may show that a certain set of natural frequencies is likely to be caused by a certain damage state. However, it is not possible, in general, to uniquely determine the damage state from a given set of natural frequencies. For example, the set of natural frequencies which is obtained when joint 1 of the lattice considered here is disconnected, would also be obtained if joint 2, 11 or 12 were disconnected. Also, as illustrated by Fig. 8, the natural frequencies associated with two damage states may be nearly identical over a particular frequency range, making an experimental distinction between the two states difficult. The set of natural frequencies, which is determined by the overall state of the lattice, cannot, in general, give information about the state of a particular section of the lattice. Thus, measurement of natural frequencies alone cannot give a totally quantitative assessment of structural integrity, since such an assessment requires, in general, the location of damage within the lattice.

Determination of damage location in lattice structures may be possible by analyzing propagating disturbances or waves. If a disturbance is introduced at a given point in a lattice structure and monitored at a nearby point, the initial portion of the detected disturbance at the monitored point depends only on the local state of the lattice. Extraction of information from detected disturbances requires an understanding of wave propagation in lattice structures. Analysis of wave propagation in lattice structures is discussed in [3] and [4]. Detection and analysis of wave propagation is more difficult than measurement of natural frequencies, but such analysis may lead to NDE methods capable of providing more complete assessments of structural integrity.

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[1] J.H. Williams, Jr. and R.J. Nagem, "Computation of Natural Frequencies of Planar Lattice Structure", AFOSR Technical Report, March 1987.

[2] M. Richardson and R. Potter, "Identification of the Modal Properties of an Elastic Structure from Measured Transfer Function Data", Instrument Society of America, ISA ASI 74250, 1974, pp. 239-246.

[3] J.H. Williams, Jr., H.K. Yeung and R.J. Nagem," Joint Coupling Matrices for Dynamic Analysis of Large Space Structures", AFOSR Technical Report, April 1986.

[4] J.H. Williams, Jr. and R.J. Nagem, "Wave-Mode Coordinates and Scattering Matrices for Wave Propagation in Large Space Structures", AFOSR Technical Report, October 1986.

TABLE 1 First twenty-five nonzero natural frequencies of undamaged lattice.

Flexible Mode Number	Frequency (rad/sec)
1	308.5
2	433.5
3	606.5
4	726.5
5	932.5
6	1438.5
7	1535.5
8	1766.5
9	2071.5
10	2115.5
11	2307.5
12	2479.5
13	2887.5
14	2979.5
15	3242.5
16	3249.5
17	3254.5
18	3257.5
19	3258.5
20	3552.5
21	4120.5
22	5923.5
23	6094.5
24	6704.5
25	6894.5

TABLE 2 First twenty-five nonzero natural frequencies associated with damage state 1.

Flexible Mode Number	Frequency (rad/sec)
1	162.5
2	319.5
3	407.5
4	442.5
5	574.5
6	714.5
7	919.5
8	1465.5
9	1642.5
10	1913.5
11	2081.5
12	2199.5
13	2362.5
14	2587.5
15	2851.5
16	3080.5
17	3243.5
18	3251.5
19	3256.5
20	3258.5
21	3311.5
22	3626.5
23	4164.5
24	5960.5
25	6291.5

TABLE 3 First twenty-five nonzero natural frequencies associated with damage state 2.

Flexible Mode Number	Frequency (rad/sec)
1	68.5
2	138.5
3	353.5
4	383.5
5	415.5
6	468.5
7	621.5
8	664.5
9	895.5
10	1477.5
11	1713.5
12	1862.5
13	2125.5
14	2167.5
15	2417.5
16	2687.5
17	2862.5
18	3129.5
19	3245.5
20	3254.5
21	3258.5
22	3271.5
23	3353.5
24	3522.5
25	4097.5

TABLE 4 First twenty-five nonzero natural frequencies associated with damage state 3.

Flexible Mode Number	Frequency (rad/sec)
1	207.5
2	412.5
3	417.5
4	482.5
5	571.5
6	752.5
7	952.5
8	1456.5
9	1594.5
10	1847.5
11	2028.5
12	2169.5
13	2378.5
14	2442.5
15	2905.5
16	2954.5
17	3228.5
18	3245.5
19	3254.5
20	3258.5
21	3493.5
22	3874.5
23	4284.5
24	5934.5
25	6176.5

TABLE 5 First twenty-five nonzero natural frequencies associated with damage state 4.

Flexible Mode Number	Frequency (rad/sec)
1	74.5
2	319.5
3	383.5
4	469.5
5	629.5
6	730.5
7	895.5
8	1464.5
9	1512.5
10	1758.5
11	2095.5
12	2159.5
13	2471.5
14	2621.5
15	2729.5
16	2929.5
17	3245.5
18	3254.15
19	3254.25
20	3258.5
21	3342.5
22	3522.5
23	4096.5
24	5981.5
25	6139.5

TABLE 6 First twenty-five nonzero natural frequencies associated with damage state 5.

Flexible Mode Number	Frequency (rad/sec)
1	51.5
2	148.5
3	400.5
4	403.5
5	432.5
6	499.5
7	580.5
8	750.5
9	856.5
10	1504.5
11	1575.5
12	1874.5
13	2109.5
14	2358.5
15	2415.5
16	2567.5
17	2854.5
18	3097.5
19	3248.5
20	3253.5
21	3256.5
22	3258.5
23	3398.5
24	3687.5
25	3985.5

TABLE 7 First twenty-five nonzero natural frequencies associated with damage state 6.

Flexible Mode Number	Frequency (rad/sec)
1	288.5
2	331.5
3	464.5
4	514.5
5	615.5
6	720.5
7	964.5
8	1460.5
9	1569.5
10	1841.5
11	1997.5
12	2207.5
13	2381.5
14	2474.5
15	2871.5
16	2951.5
17	3191.5
18	3249.5
19	3254.5
20	3257.5
21	3565.5
22	3775.5
23	4361.5
24	5940.5
25	6130.5

TABLE 8 First twenty-five nonzero natural frequencies associated with damage state 7.

Flexible Mode Number	Frequency (rad/sec)
1	62.5
2	299.5
3	427.5
4	488.5
5	608.5
6	753.5
7	861.5
8	1466.5
9	1520.5
10	1821.5
11	1966.5
12	2206.5
13	2404.5
14	2537.5
15	2817.5
16	3022.5
17	3198.5
18	3249.15
19	3249.35
20	3257.25
21	3257.35
22	3663.5
23	4012.5
24	5981.5
25	6110.5

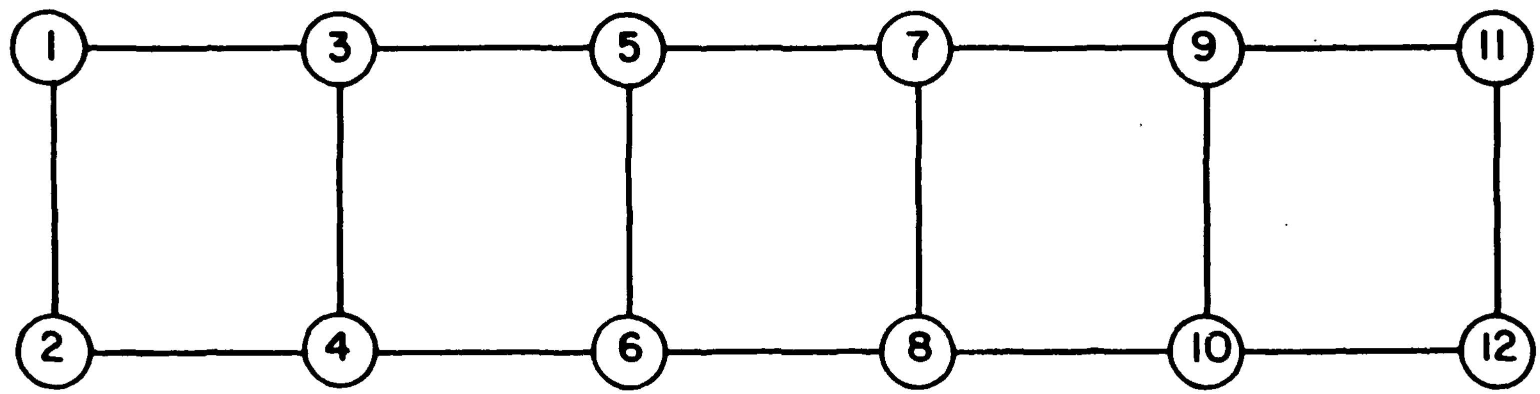

Fig. 1 Lattice model.

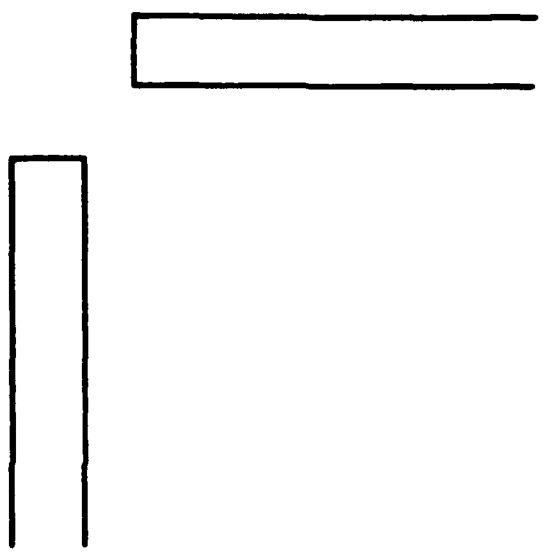

Fig. 2 Disconnected joint associated with damage state 1, joint 1.

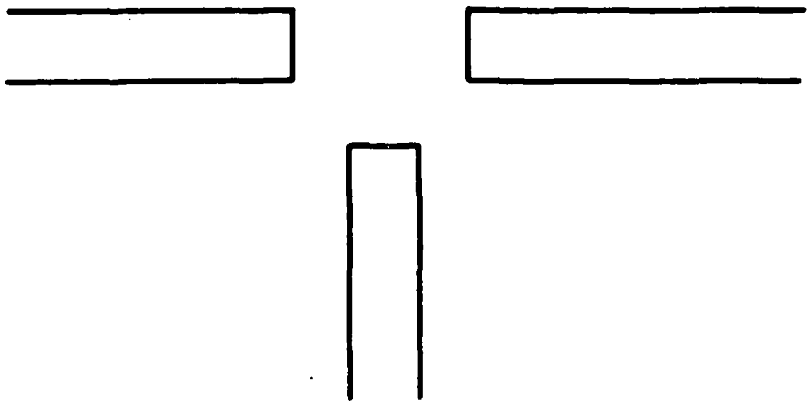

Fig. 3 Disconnected joint associated with damage states 2 and 5, joint 3 and joint 5, respectively.

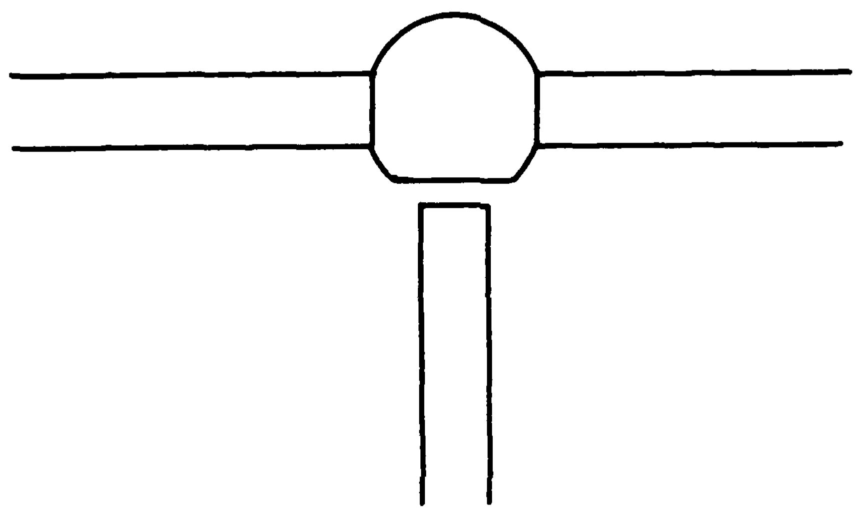

Fig. 4 Disconnected joint associated with damage states 3 and 6, joint 3 and joint 5, respectively.

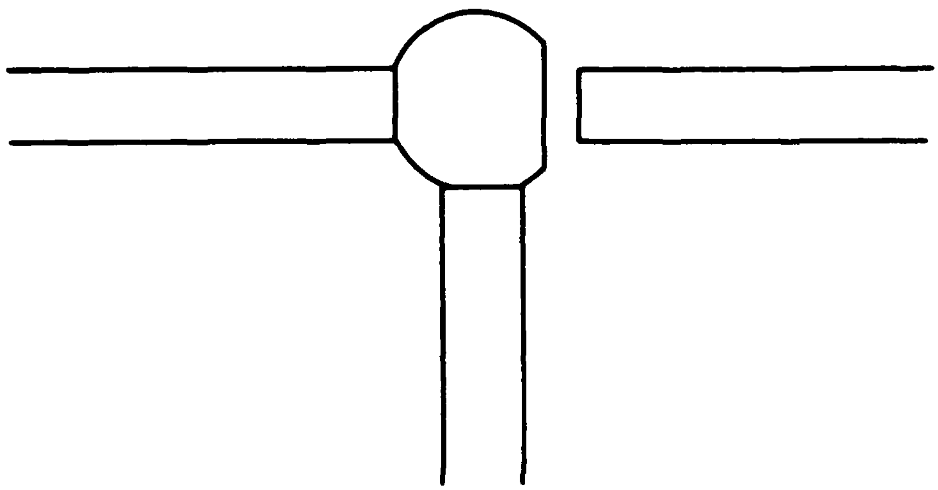

Fig. 5 Disconnected joint associated with damage states 4 and 7, joint 3 and joint 5, respectively.

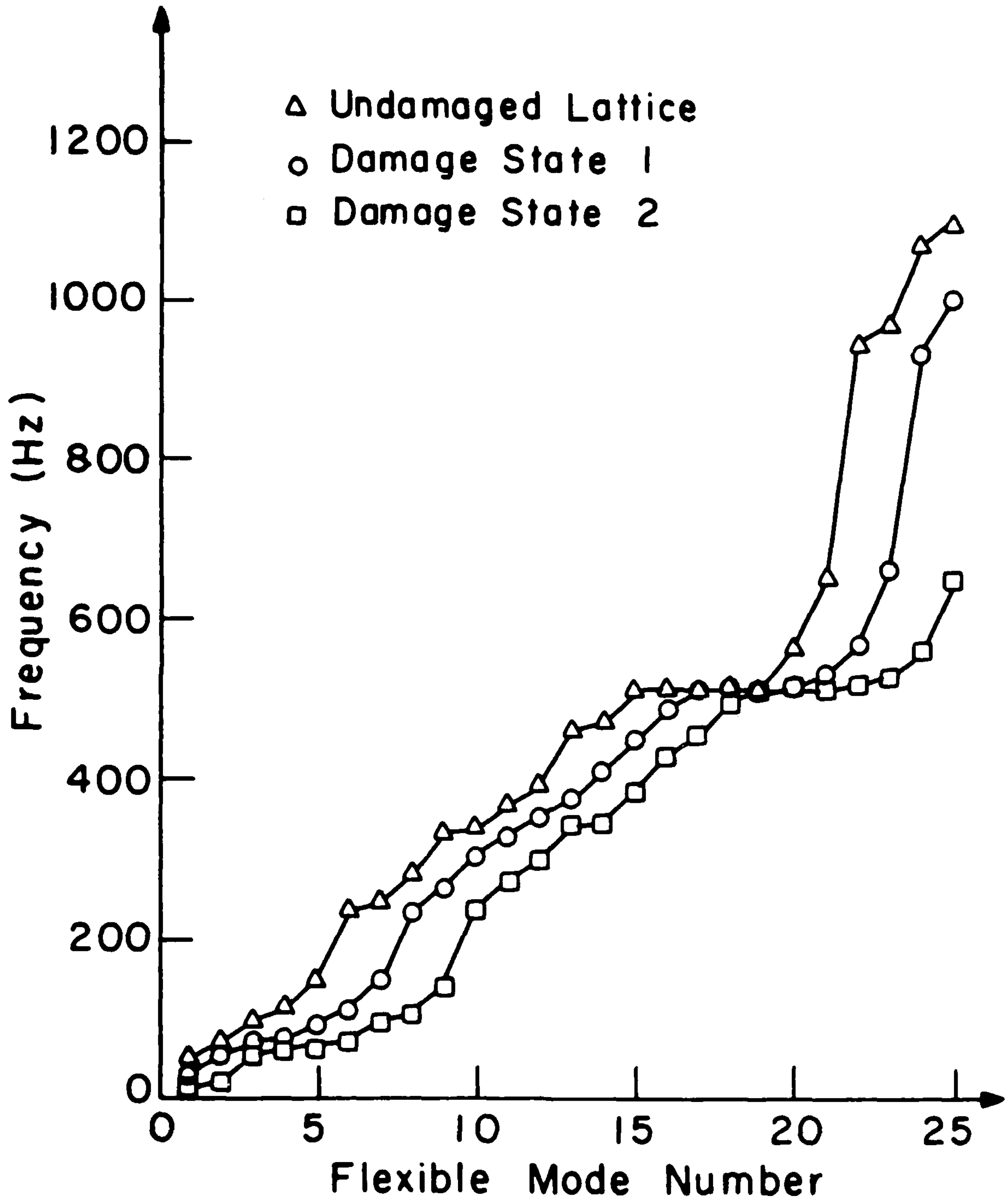

Fig. 6 Natural frequencies of undamaged lattice and natural frequencies associated with damage states 1 and 2.

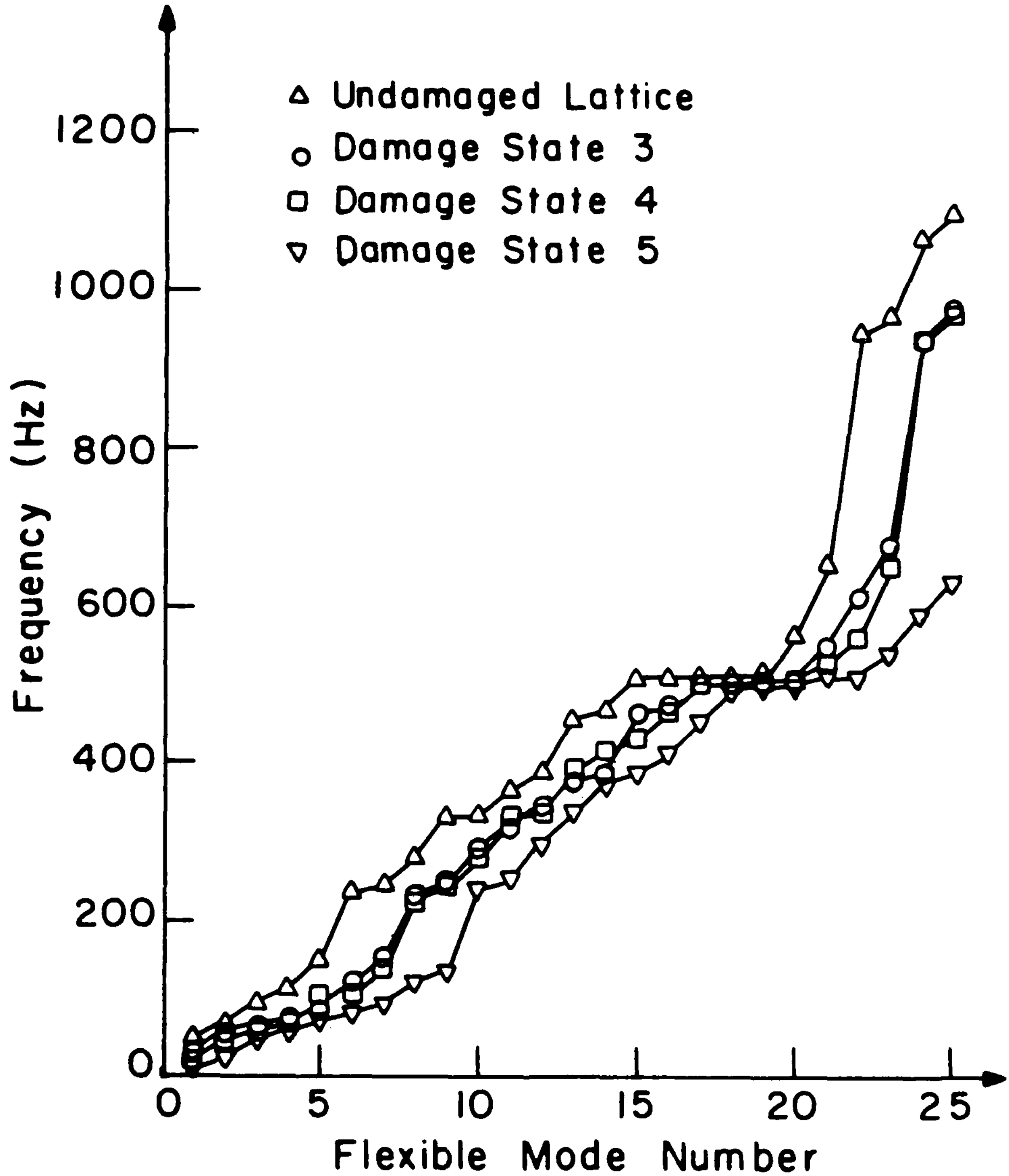

Fig. 7 Natural frequencies of undamaged lattice and natural frequencies associated with damage states 3, 4 and 5.

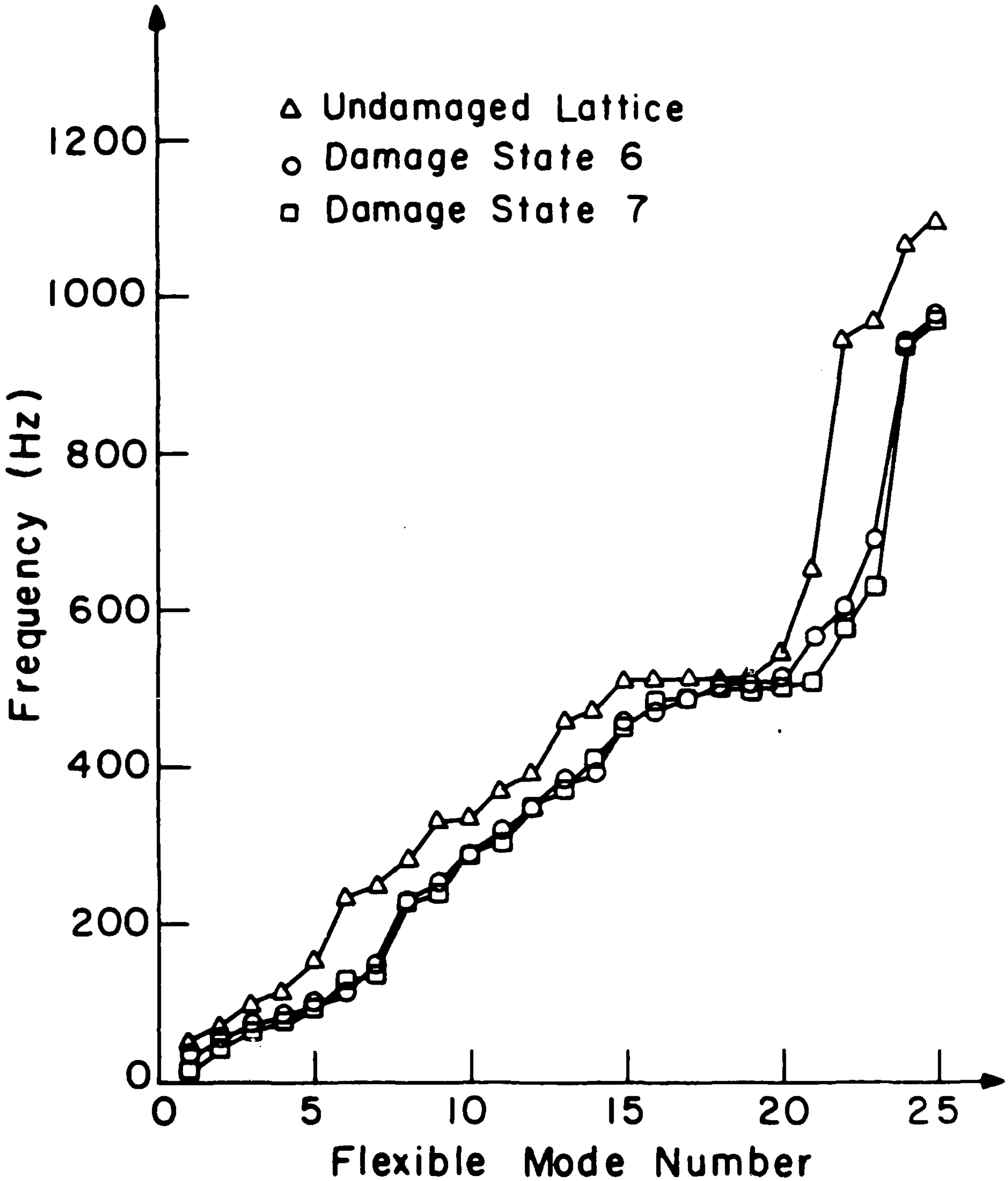

Fig. 8 Natural frequencies of undamaged lattice and natural frequencies associated with damage states 6 and 7.

APPENDIX A: JOINT COUPLING MATRICES FOR DISCONNECTED AND PARTIALLY DISCONNECTED JOINTS

In this appendix, joint coupling matrices for disconnected and partially disconnected two-dimensional L and T-joints are derived. It is assumed that the state vectors of the one-dimensional members which meet at the joints are of the form

$$\underline{z}(x,t) = \begin{Bmatrix} u(x,t) \\ v(x,t) \\ \psi(x,t) \\ M(x,t) \\ V(x,t) \\ F(x,t) \end{Bmatrix} \qquad \text{(A1)}$$

where u(x,t) is the longitudinal displacement of the member, v(x,t) is the transverse displacement of the member, $\psi(x,t)$ is the rotation of the member, M(x,t) is the bending moment in the member, V(x,t) is the shear force in the member, F(x,t) is the axial force in the member, x is a spatial coordinate which extends along the length of the member and t is time. The components of the state vector and the sign convention adopted here for the components of the state vector are shown in Fig. A1. Throughout this appendix, an overbar will denote a Fourier transform.

Disconnected L-joint

A disconnected L-joint is shown in Fig. A2(a). The components of the state vectors of the members which meet at the joint are shown in Fig. A2(b).

Since the joint is disconnected, it is assumed that points 1 and 2 are free ends. Therefore,

$$M_1 = 0 \tag{A2}$$

$$V_1 = 0 \tag{A3}$$

$$F_1 = 0 \tag{A4}$$

$$M_2 = 0 \tag{A5}$$

$$V_2 = 0 \tag{A6}$$

$$F_2 = 0 \tag{A7}$$

After taking a Fourier transform, eqns. (A2) through (A7) can be written as

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \bar{u}_1 \\ \bar{v}_1 \\ \bar{\psi}_1 \\ \bar{M}_1 \\ \bar{V}_1 \\ \bar{F}_1 \end{array} \right\} \\ \left\{ \begin{array}{c} \bar{u}_2 \\ \bar{v}_2 \\ \bar{\psi}_2 \\ \bar{M}_2 \\ \bar{V}_2 \\ \bar{F}_2 \end{array} \right\} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right\} \tag{A8}$$

The 6 x 12 matrix in eqn. (A8) is the joint coupling matrix of the disconnected L-joint of Fig. A2(a), although the matrix does not, in fact, couple the two members at all.

Disconnected T-joint

A disconnected T-joint is shown in Fig. A3(a). The components of the state vectors of the members which meet at the joint are shown in Fig. A3(b). Since the joint is disconnected, it is assumed that points 1, 2 and 3 are free ends. Therefore,

$$M_1 = 0 \tag{A9}$$

$$V_1 = 0 \tag{A10}$$

$$F_1 = 0 \tag{A11}$$

$$M_2 = 0 \tag{A12}$$

$$V_2 = 0 \tag{A13}$$

$$F_2 = 0 \tag{A14}$$

$$M_3 = 0 \tag{A15}$$

$$V_3 = 0 \tag{A16}$$

$$F_3 = 0 \tag{A17}$$

After taking a Fourier transform, eqns. (A9) through (A17) can be written as

$$
\begin{bmatrix}
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
\left\{\begin{matrix}
\left\{\begin{matrix}\bar{u}_1\\ \bar{v}_1\\ \bar{\psi}_1\\ \bar{M}_1\\ \bar{V}_1\\ \bar{F}_1\end{matrix}\right\}\\
\left\{\begin{matrix}\bar{u}_2\\ \bar{v}_2\\ \bar{\psi}_2\\ \bar{M}_2\\ \bar{V}_2\\ \bar{F}_2\end{matrix}\right\}\\
\left\{\begin{matrix}\bar{u}_3\\ \bar{v}_3\\ \bar{\psi}_3\\ \bar{M}_3\\ \bar{V}_3\\ \bar{F}_3\end{matrix}\right\}
\end{matrix}\right\}
=
\left\{\begin{matrix}0\\0\\0\\0\\0\\0\\0\\0\\0\end{matrix}\right\}
\qquad \text{(A18)}
$$

The 9 x 18 matrix in eqn. (A18) is the joint coupling matrix of the disconnected T-joint of Fig. A3(a).

Partially Disconnected T-joint, Case 1

A partially disconnected T-joint is shown in Fig. A4(a). The components of the state vectors of the members which meet at the joint are shown in Fig. A4(b). In Fig. A4(a), it is assumed that point 3 is a free end, and that points 1 and 2 are connected by a rigid massless joint with no spatial extent. Since point 3 is a free end,

$$M_3 = 0 \tag{A19}$$

$$V_3 = 0 \tag{A20}$$

$$F_3 = 0 \tag{A21}$$

The equilibrium requirements for the joint connecting points 1 and 2 are

$$M_1 - M_2 = 0 \tag{A22}$$

$$V_1 - V_2 = 0 \tag{A23}$$

$$F_1 - F_2 = 0 \tag{A24}$$

The compatibility requirements for the joint connecting points 1 and 2 are

$$u_1 - u_2 = 0 \tag{A25}$$

$$v_1 - v_2 = 0 \tag{A26}$$

$$\psi_1 - \psi_2 = 0 \tag{A27}$$

After taking a Fourier transform, eqns. (A19) through (A27) can be written as

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\left\{ \begin{matrix}
\left\{ \begin{matrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{\psi}_1 \\ \bar{M}_1 \\ \bar{V}_1 \\ \bar{F}_1 \end{matrix} \right\} \\
\left\{ \begin{matrix} \bar{u}_2 \\ \bar{v}_2 \\ \bar{\psi}_2 \\ \bar{M}_2 \\ \bar{V}_2 \\ \bar{F}_2 \end{matrix} \right\} \\
\left\{ \begin{matrix} \bar{u}_3 \\ \bar{v}_3 \\ \bar{\psi}_3 \\ \bar{M}_3 \\ \bar{V}_3 \\ \bar{F}_3 \end{matrix} \right\}
\end{matrix} \right\}
=
\left\{ \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \right\}
\qquad \text{(A28)}
$$

The 9 x 18 matrix in eqn. (A28) is the joint coupling matrix for the partially disconnected T-joint of Fig. A4(a).

Partially Disconnected T-joint, Case 2

A partially disconnected T-joint is shown in Fig. A5(a). The components of the state vectors of the members which meet at the joint are shown in Fig. A5(b). In Fig. A5(a), it is assumed that point 2 is a free end, and that points 1 and 3 are connected by a rigid massless joint with no spatial extent. Since point 2 is a free end,

$$M_2 = 0 \tag{A29}$$

$$V_2 = 0 \tag{A30}$$

$$F_2 = 0 \tag{A31}$$

The equilibrium requirements for the joint connecting points 1 and 3 are

$$M_1 - M_3 = 0 \tag{A32}$$

$$V_1 - F_3 = 0 \tag{A33}$$

$$F_1 + V_3 = 0 \tag{A34}$$

The compatibility requirements for the joint connecting points 1 and 3 are

$$u_1 - v_3 = 0 \tag{A35}$$

$$v_1 + u_3 = 0 \tag{A36}$$

$$\psi_1 - \psi_3 = 0 \tag{A37}$$

After taking a Fourier transform, eqns. (A29) through (A37) can be written as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \left\{ \begin{matrix} \left\{ \begin{matrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{\psi}_1 \\ \bar{M}_1 \\ \bar{V}_1 \\ \bar{F}_1 \end{matrix} \right\} \\ \left\{ \begin{matrix} \bar{u}_2 \\ \bar{v}_2 \\ \bar{\psi}_2 \\ \bar{M}_2 \\ \bar{V}_2 \\ \bar{F}_2 \end{matrix} \right\} \\ \left\{ \begin{matrix} \bar{u}_3 \\ \bar{v}_3 \\ \bar{\psi}_3 \\ \bar{M}_3 \\ \bar{V}_3 \\ \bar{F}_3 \end{matrix} \right\} \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \right\} \qquad \text{(A38)}$$

The 9 x 18 matrix in eqn. (A38) is the joint coupling matrix for the partially disconnected T-joint of Fig. A5(a).

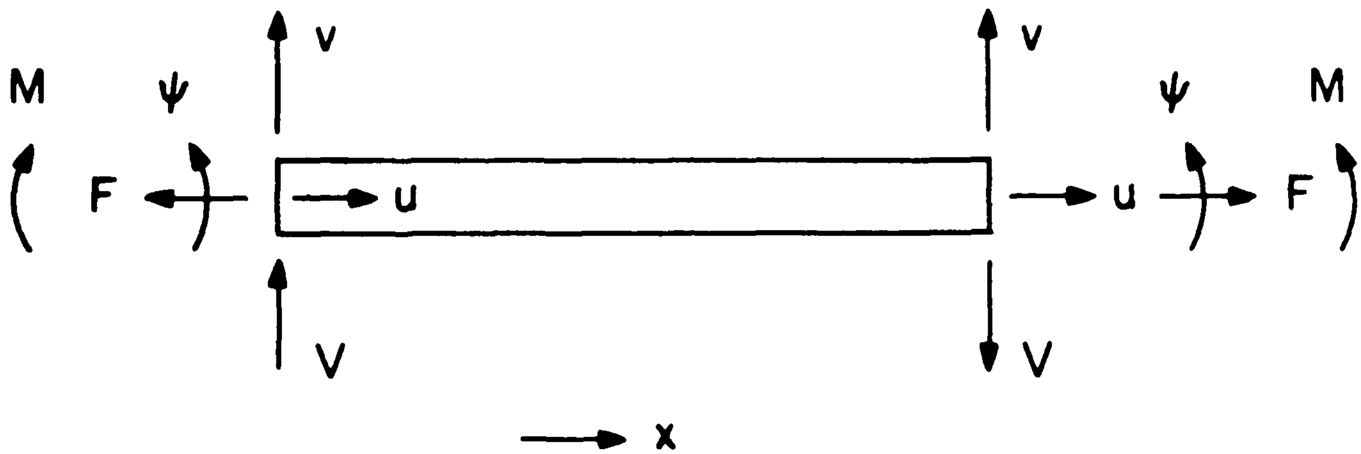

Fig. A1 Lattice member, showing components of state vectors and sign convention.

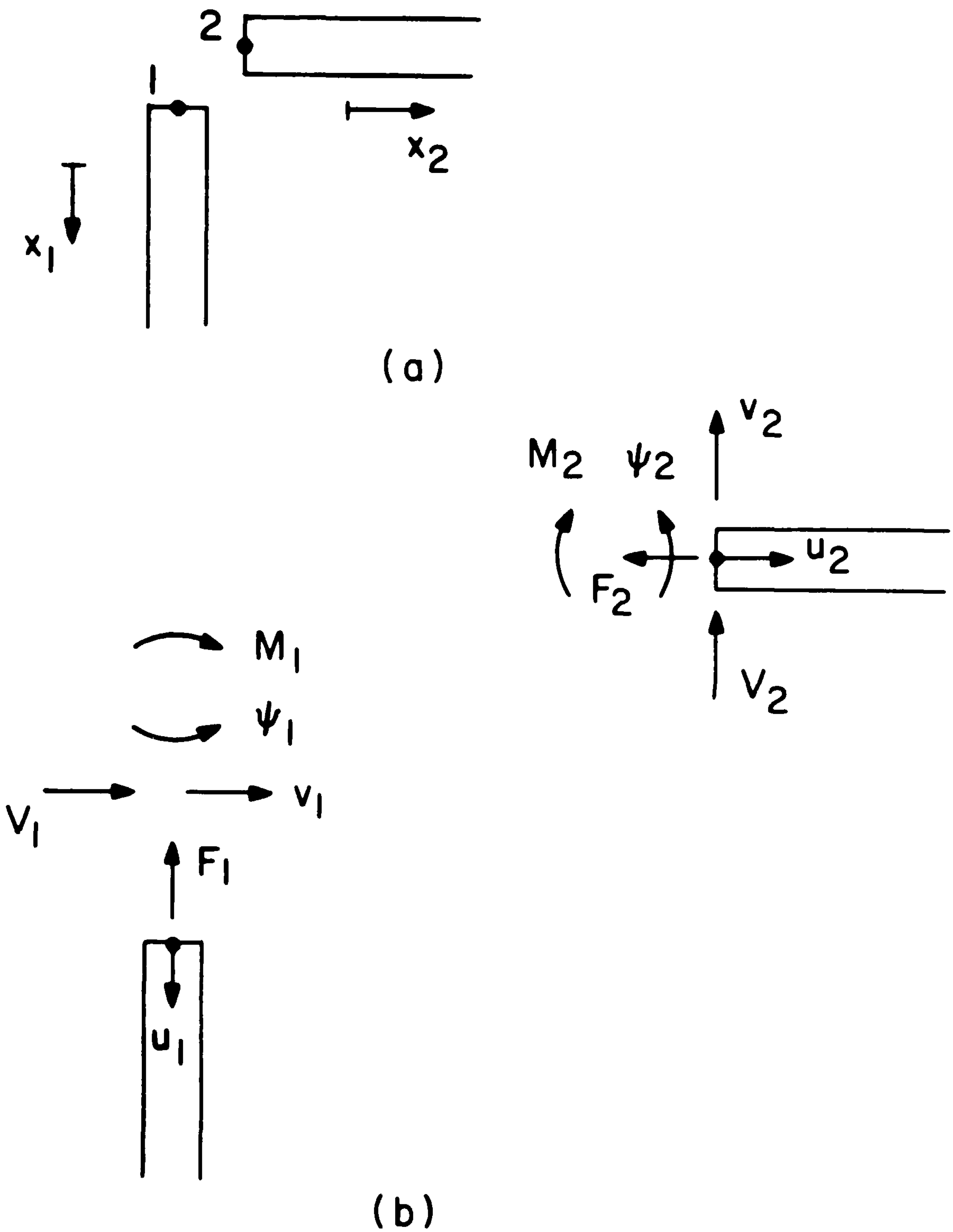

Fig. A2 Disconnected L-joint.

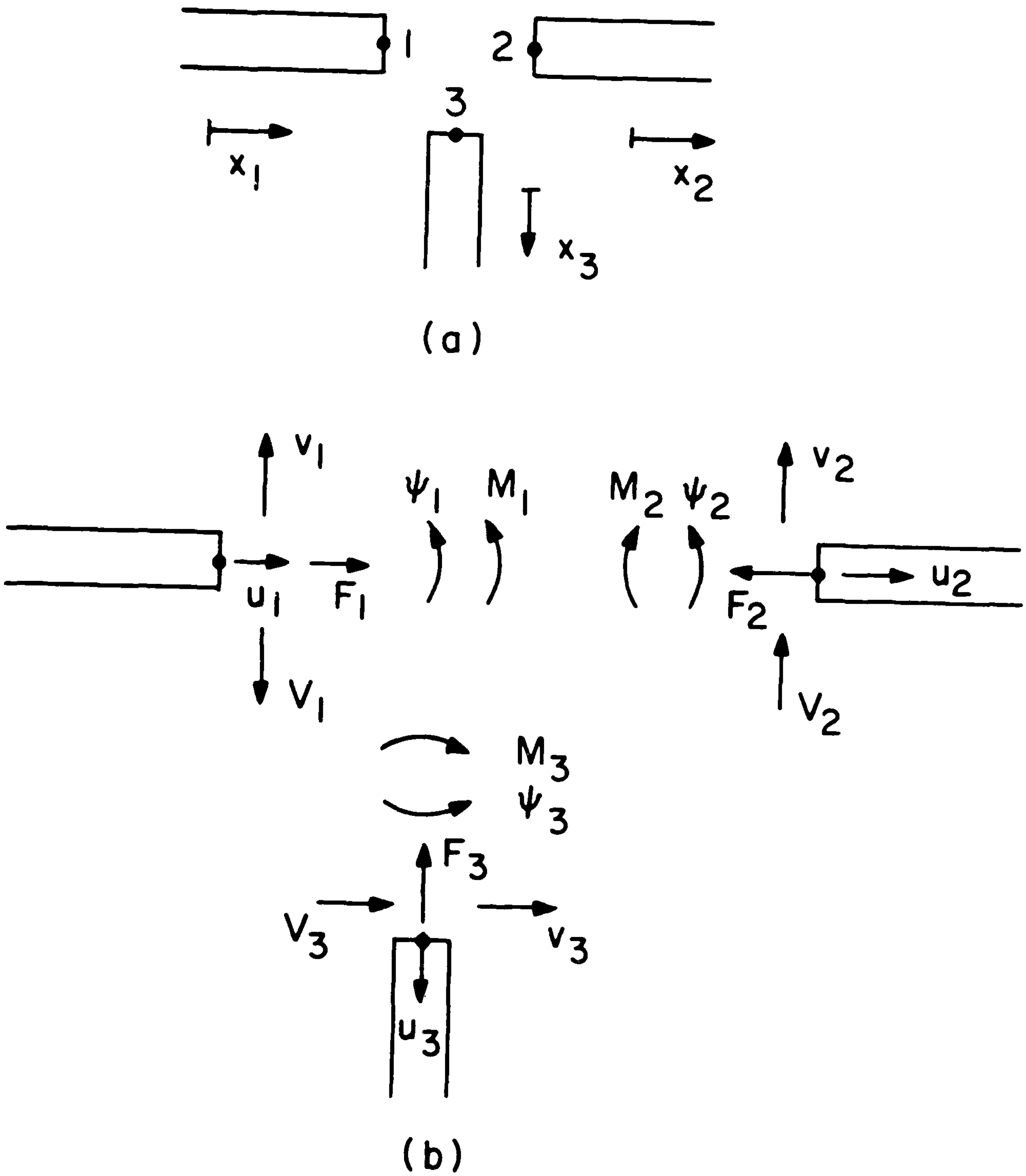

Fig. A3 Disconnected T-joint.

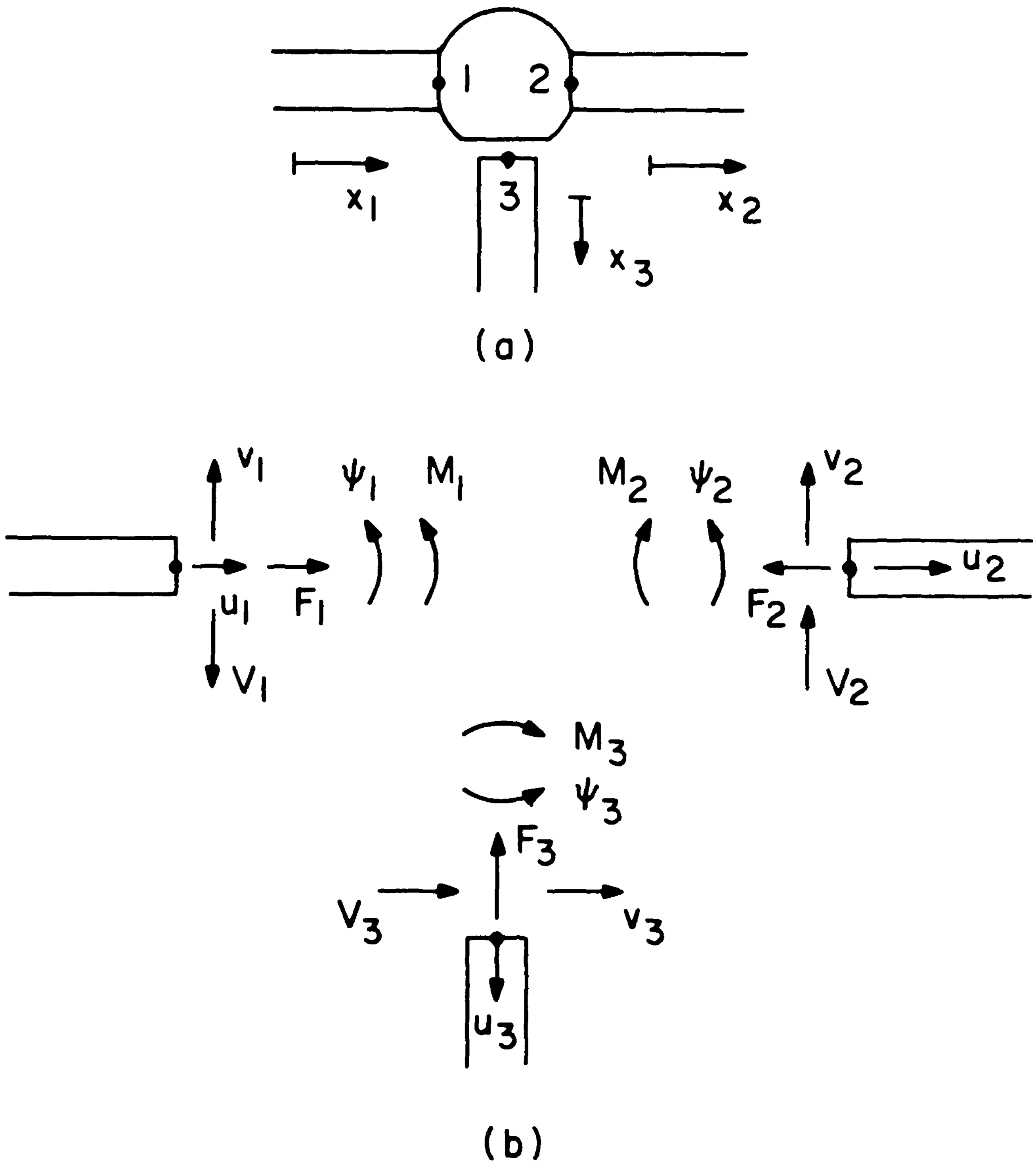

Fig. A4 Partially disconnected T-joint.

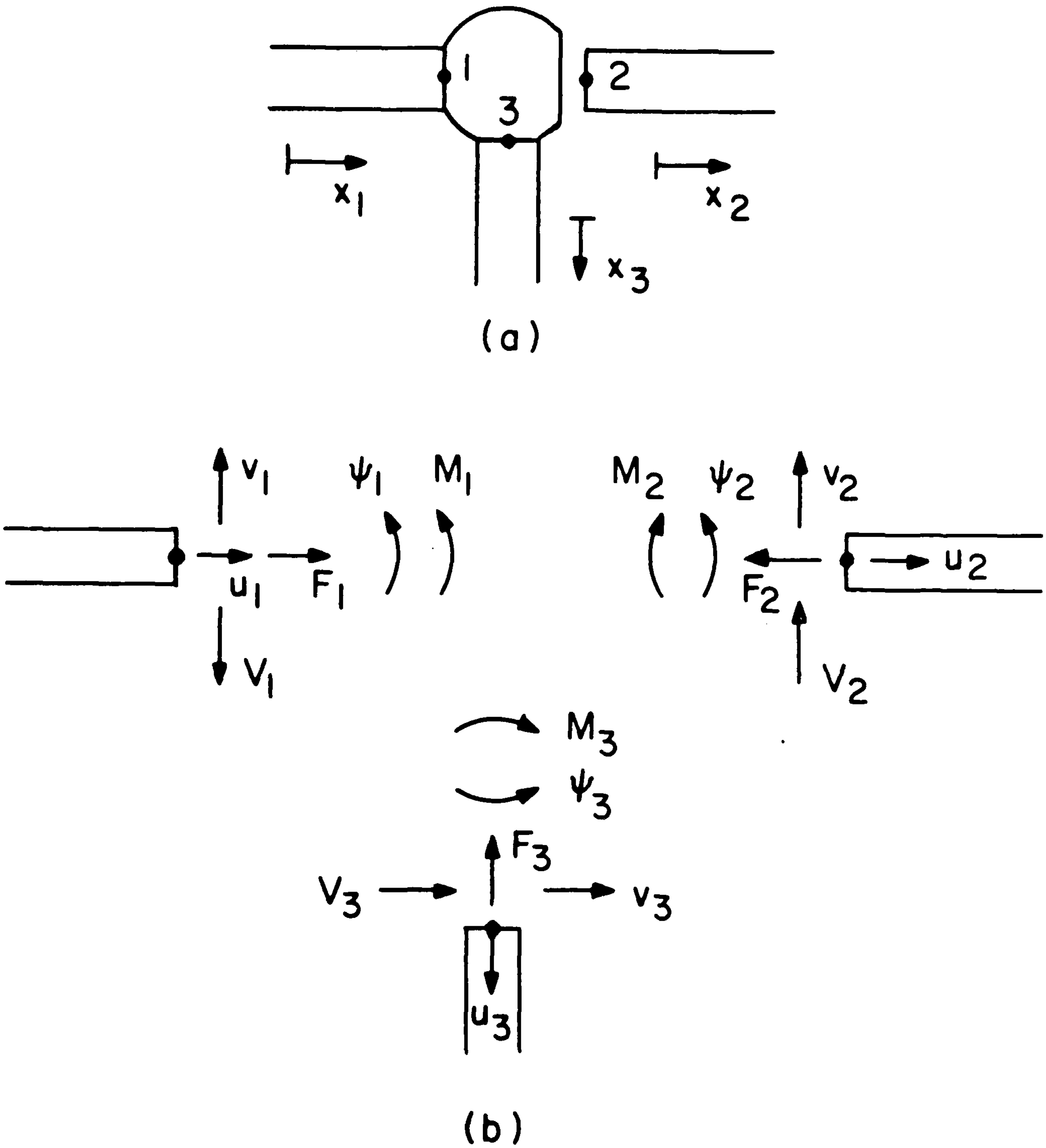

Fig. A5 Partially disconnected T-joint.

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